MODEL ANSWERS

Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
Further Statistics 1

Sample Assessment Material for first teaching September 2017
Time: 50 minutes

Paper Reference
8FM0/2F

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 4 questions in this question paper. The total mark for this paper is 40.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. A university foreign language department carried out a survey of prospective students to find out which of three languages they were most interested in studying.

A random sample of 150 prospective students gave the following results.

		Language			
		French	**Spanish**	**Mandarin**	**Total**
Gender	**Male**	23	22	20	65
	Female	38	32	15	85
	Total	61	54	35	150

A test is carried out at the 1% level of significance to determine whether or not there is an association between gender and choice of language.

(a) State the null hypothesis for this test.

(1)

(b) Show that the expected frequency for females choosing Spanish is 30.6

(1)

(c) Calculate the test statistic for this test, stating the expected frequencies you have used.

(3)

(d) State whether or not the null hypothesis is rejected. Justify your answer.

(2)

(e) Explain whether or not the null hypothesis would be rejected if the test was carried out at the 10% level of significance.

(1)

a) H_0: Language and gender are independent (no association)

b) $\frac{54 \times 85}{150} = 30.6$

Total taking Spanish = 54

Total women = 85

Question 1 continued

c) $\frac{61\times65}{150} = 26.4\dot{3}$ $\quad \frac{54\times65}{150} = 23.4$

$\frac{35\times65}{150} = 15.1\dot{6}$ $\quad \frac{61\times85}{150} = 34.5\dot{6}$

$\frac{54\times85}{150} = 30.6$ $\quad \frac{35\times85}{150} = 19.8\dot{3}$

O_i	E_i	$\frac{(O_i-E_i)^2}{E_i}$
23	$26.4\dot{3}$	0.4459
22	23.4	0.0838
20	$15.1\dot{6}$	1.5403
38	$34.5\dot{6}$	0.3410
32	30.6	0.0641
15	$19.8\dot{3}$	1.1779

$\chi^2 = \sum_1 \frac{(O_i-E_i)^2}{E_i} = 3.653$ (3dp)

d) degree of freedom = $(\text{n.o cells rows} - 1)(\text{n.o cells column} - 1)$

$= (2-1)(3-1) = 2$

$\chi^2_2(0.01) = 9.210$

$3.653 < 9.210$

$\therefore$ insufficient evidence to reject H_0.
Language and gender are independent (no association).

Question 1 continued

e) $\chi^2_2(0.1) = 4.605$

$3.653 < 4.605$ so H_0 still not rejected

Question 1 continued

(Total for Question 1 is 8 marks)

2. The discrete random variable X has probability distribution given by

x	–1	0	1	2	3
$P(X = x)$	c	a	a	b	c

The random variable $Y = 2 - 5X$

Given that $E(Y) = -4$ and $P(Y \geqslant -3) = 0.45$

(a) find the probability distribution of X.

(7)

Given also that $E(Y^2) = 75$

(b) find the exact value of $\text{Var}(X)$

(2)

(c) Find $P(Y > X)$

(2)

a) $Y = 2 - 5X$

$E(Y) = 2 - 5E(X)$

$-4 = 2 - 5E(X)$

$5E(X) = 6$

$E(X) = 1.2$

$E(X) = \sum x P(x)$

$= -c + (0 \times a) + a + 2b + 3c$

$= 1.2$

$\Rightarrow a + 2b + 2c = 1.2$ — (1)

$P(Y \geq -3) = 0.45$

$\therefore P(2 - 5X \geq -3) =$

$P(X \leq 1) = 0.45$

$P(x = -1) + P(X = 0) + P(X = 1)$

$= 2a + c = 0.45$ — (2)

Probability sums to 1:

$2a + b + 2c = 1$ — (3)

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Question 2 continued

Using Simultaneous equations:

③ − ② : $b + c = 0.55$

$2b + 2c = 1.1$ — ④

Sub ④ into ① :

$a + 1.1 = 1.2$

$\therefore a = 0.1$

$\therefore$ ③ when $a = 0.1$:

$(2 \times 0.1) + b + 2c = 1$

$b + 2c = 0.8$

④ − ③ : $(2b + 2c) - (b + 2c) = 1.1 - 0.8$

$b = 0.3$

$\therefore$ ③ when $a = 0.1$ and $b = 0.3$:

$(2 \times 0.1) + 0.3 + 2c = 1$

$2c = 0.5$

$c = 0.25$

$a = 0.1$, $b = 0.3$, $c = 0.25$

b) $\text{Var}(Y) = E(Y^2) - (E(Y))^2$

so $\text{Var}(Y) = 75 - (-4)^2$

$= 59$

$Y = aX + b \Rightarrow \text{Var}(Y) = a^2 \text{Var}(X)$

$\text{Var}(Y) = 5^2 \text{Var}(X) \therefore \text{Var}(X) = 2.36$

Question 2 continued

c) $P(Y > X) = P(2 - 5X > X) =$

$= P(X < \frac{1}{3})$

$= a + c$

$= 0.1 + 0.25$

$= 0.35$

Question 2 continued

(Total for Question 2 is 11 marks)

3. Two car hire companies hire cars independently of each other.

Car Hire *A* hires cars at a rate of 2.6 cars per hour.

Car Hire *B* hires cars at a rate of 1.2 cars per hour.

(a) In a 1 hour period, find the probability that each company hires exactly 2 cars.

(2)

(b) In a 1 hour period, find the probability that the total number of cars hired by the two companies is 3

(2)

(c) In a 2 hour period, find the probability that the total number of cars hired by the two companies is less than 9

(2)

On average, 1 in 250 new cars produced at a factory has a defect.

In a random sample of 600 new cars produced at the factory,

(d) (i) find the mean of the number of cars with a defect,

(ii) find the variance of the number of cars with a defect.

(2)

(e) (i) Use a Poisson approximation to find the probability that no more than 4 of the cars in the sample have a defect.

(ii) Give a reason to support the use of a Poisson approximation.

(2)

a) $X \sim Po(2.6)$ $Y \sim Po(1.2)$

$P(\text{EACH HIRE 2 IN 1 HOUR}) = P(X=2) \times P(Y=2)$

$= 0.25104... \times 0.21685...$

INDEPENDENT

$= 0.05444...$

$= \mathbf{0.0544}$ **(3sf)**

b) DEFINE NEW VARIABLE $W = X + Y$

$W \sim Po(3.8)$

$P(W=3) = 0.20458...$

$= \mathbf{0.205}$ **(3sf)**

Question 3 continued

c) 2 HOURS → $T = 2W$

$T \sim Po(2 \times 3.8)$

$P(T < 9) = P(T \leq 8) = 0.648199\ldots = 0.648$ (3sf)

d) i. MEAN $= np = \frac{1}{250} \times 600$

$= 2.4$

ii. VAR $= np(1-p) = 2.3904$

e) i. $D \sim Po(2.4)$ EXPECTED NO. DEFECTS IN SAMPLE

$P(D \leq 4) = 0.9041\ldots$

ii. N IS LARGE & P IS SMALL

SO MEAN ≈ VARIANCE

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. The discrete random variable X follows a Poisson distribution with mean 1.4

(a) Write down the value of

(i) $P(X = 1)$

(ii) $P(X \leqslant 4)$

(2)

The manager of a bank recorded the number of mortgages approved each week over a 40 week period.

Number of mortgages approved	0	1	2	3	4	5	6
Frequency	10	16	7	4	2	0	1

(b) Show that the mean number of mortgages approved over the 40 week period is 1.4

(1)

The bank manager believes that the Poisson distribution may be a good model for the number of mortgages approved each week.

She uses a Poisson distribution with a mean of 1.4 to calculate expected frequencies as follows.

Number of mortgages approved	0	1	2	3	4	5 or more
Expected frequency	9.86	r	9.67	4.51	1.58	s

(c) Find the value of r and the value of s giving your answers to 2 decimal places.

(2)

The bank manager will test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(d) Calculate the test statistic and state the conclusion for this test. State clearly the degrees of freedom and the hypotheses used in the test.

(6)

$X \sim Po(1.4)$

a) i) $P(X=1) = 0.345123\ldots = 0.345$ (3SF)

ii) $P(X \leqslant 4) = 0.98575\ldots = 0.986$ (3SF)

b) MEAN $= \{(0\times10)+(1\times16)+(2\times7)+(3\times4)+(4\times2)+(5\times0)+(6\times1)\} \div 40$

$= 1.4$

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Question 4 continued $X \sim Po(1.4)$

c) $P(X = 1) = 0.34523\ldots$

so $r = 40 \times P(X=1) = 13.81$

$P(X > 4) = 1 - P(X \leq 4)$

so $S = 40 \times (1 - 0.98575)$

$= 0.57$

d) H_0: Poisson Distribution is a suitable model.

H_1: Poisson Distribution is not a suitable model.

IF EX. FREQ. < 5, COMBINE CELLS

SO COMBINE LAST 3 CELLS

LAST 3 CELLS

Perform Chi squared test:

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(10-9.86)^2}{9.86} + \ldots + \frac{(7-(4.51+1.58+0.571))^2}{(4.51+1.58+0.571)}$$

$= 1.1$

$d.o.f. = 4 - 1 - 1 = 2$

4 is the number of cells
−1 for the estimation of the mean.
−1 for the limitation of 40

Question 4 continued

$\chi^2_2(0.05) = 5.991$

$5.991 > 1.1$

$\therefore$ insufficient evidence to reject evidence to reject H_0.

No mortgages approved in a week is modelled by Poisson distribution.

Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR PAPER IS 40 MARKS